THE

INTERNATIONAL MATHEMATICAL UNION in the DEVELOPING WORLD: Past, Present and Future

White paper

2014

Commission for Developing Countries,
International Mathematical Union

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This paper offers a broad view of the role of the International Mathematical Union (IMU) in supporting the progress of mathematics and advanced mathematical education in developing nations – its purpose, history, current activities, and some options for future action. In doing so, the document seeks to make three primary points:

(1) Vibrant communities of mathematical research are vital to the scientific, technological, social, and economic development of every nation. Because much of what mathematicians do may not be understood by the general public, the IMU and its constituencies seek to enhance this vitality through broadly communicating the benefits of mathematics.

(2) Quality mathematics education for all is the foundation for stronger mathematics worldwide – and a principal goal of the IMU. To strengthen this foundation, the teaching and learning of mathematics should keep pace with the needs of modern societies, especially those in the developing world.

(3) The IMU and its more than 70 member countries are committed to expanding mathematical capacity that is based on three platforms: access to quality mathematics education for all; an emphasis on nurturing mathematical talent and rewarding accomplishment wherever they appear, especially in developing countries; and provide open access to the full richness of historical and current mathematics literature.

* This white paper is intended for policy makers, funding agencies, constituencies of the IMU and ICMI, and for others who would like to learn more about the activities and objectives of the IMU. It was written for the *Friends of the IMU* (FIMU) by editorial consultant Alan H. Anderson, after extensive discussions with members of the Executive Committee and of other IMU committees, the IMU constituencies, and others, and has been approved by the Board of FIMU.

Executive Summary

The International Mathematical Union (IMU) is an international non-governmental, non-profit scientific organization. It is a member of the International Council of science (ICSU), an umbrella organization for all scientific unions. IMU was created in 1920, and its members are countries. The **main objectives** of IMU are:

• To promote international cooperation in mathematics;

• To support and assist the International Congress of Mathematicians (ICM) and to award the IMU Prizes: the Fields medals, the Nevanlinna and Gauss Prizes, and the Chern medal. These quadrennial activities celebrate the major achievements in mathematical sciences;

• To encourage and support other international mathematical activities considered likely to contribute to the development of mathematical sciences in any of its aspects – pure, applied, or educational;

• A special focus is to support the development of mathematical research and education in the developing countries, by helping training of high qualified personal, networking and capacity building both in education and research.

The IMU has four commissions:

• The Commission for Developing Countries (CDC),

• The International Commission for Mathematical Instruction (ICMI),

• The Committee for Electronic Information and Communication,

• The International Commission on the History of Mathematics.

The first two are the most important for this report.

The **Commission for Developing Countries (CDC)** was created in 2006 from the former Committee for Development of Exchange (CDE), corresponding to the will of IMU of increasing its activities in the developing countries. The IMU works in deep partnership with CDC and ICMI on these issues.

The **International Commission for Mathematical Instruction (ICMI)** was created in 1908, 12 years before IMU. ICMI is as large as all the other parts of IMU together. It has an independent administration with its own General Assembly and its own quadrennial congress, the International Congress on Mathematical Education (ICME).

Since 2011, IMU has a permanent executive office in Berlin, for which Germany made a generous contribution. Due to this contribution as well as successful fund-raising activities, IMU has been able to increase its activities in the developing countries in the past few years. The needs are enormous, and actions in this direction are a priority for the coming years.

The CDC supports a wide range of projects including:

- **Volunteer Lecturer Program (VLP)**: the VLP, with help from CIMPA of France and the U.S. National Committee on Mathematics, sends lecturers to give intensive 3- to 4-week courses in mathematics at the advanced undergraduate or master's level;
- **IMU-Simons Foundation Travel Fellowship**: this program supports research visits of mathematicians in the developing world to a center of excellence for collaborative research in mathematics;
- **Abel Visiting Scholar Program** supported by the Niels Henrik Abel Board to offer young mathematicians from the developing world a research sabbatical in a center of excellence.
- **The African Mathematics Millennium Science Initiative (AMMSI)**: AMMSI administers

a scholarship program for mathematics graduate students on the African continent and helps to organize workshops, conferences, and lectures. It was initiated by the Mellon Foundation and the Science Initiative Group, based in Princeton, NJ.
- **Mentoring African Research in Mathematics (MARM)**: IMU and AMMSI assisted the London Mathematical Society in founding the MARM program, which supports mathematics and its teaching in the countries of sub-Saharan Africa. The program is financially supported by the Nuffield Foundation and the Leverhulme Trust.
- **Mathematics Library Assistance Scheme for Developing Countries**: The CDC offers to pay the shipment costs when individual scientists or institutions wish to donate books in the mathematical sciences to libraries in developing countries.
- **Adopt a Graduate Student**: This new program will be launched at ICM2014. The pilot design, initiated by the Friends of IMU, aims to match interested donors, one-on-one, with talented mathematics graduate students at a university in the developing world in need of a graduate assistantship to continue their studies.
- **Conference support** program for conferences in developing countries and for support of mathematicians of developing countries to go to international conferences.

The **NANUM project** supported by Korea allows for 1000 mathematicians from developing countries to participate in ICM2014 and benefit from satellite activities, including MENAO and the IMU General Assembly, for further networking. While IMU activities in the developing world are limited by its small budget, it collaborates with many other initiatives, including the Africa Mathematics Project, funded by the Simons Foundation.

IMU and ICMI have strengthened their linkages and synchronized their missions, with a mutual focus on the developing world. An important achievement is the **Capacity and Networking Project (CANP)**, supported by UNESCO, with the goal to enhance mathematics education in different regions of the world. There have been CANP workshops in Mali (Francophone Africa, 2011), Costa Rica (Latin America and the Caribbean, 2012), Cambodia (East Asia, 2013) and the next CANP will be in Tanzania in September 2014 (English speaking Africa). Other joint projects of IMU and ICMI include mathematical exhibitions (**Experiencing Mathematics** and the **Mathematics of Planet Earth Exhibition**), and the **Klein project** for secondary school teachers.

The **World Digital Mathematical Library (WDML)** wishes to improve the access to mathematical literature everywhere in the world.

Three primary points are guiding IMU future actions:

1. Vibrant communities of mathematical research are vital to the scientific, technological, social, and economic development of every nation; it is essential to foster such communities and to broadly communicate the benefits of mathematics to society.

2. Quality mathematics education for all is the foundation for stronger mathematics worldwide; the teaching and learning of mathematics should keep pace with the needs of modern societies, especially those in the developing world.

3. The IMU and its more than 70 member countries are committed to expanding mathematical capacity that is based on three platforms: access to quality mathematics education for all; an emphasis on nurturing mathematical talent and rewarding accomplishment wherever they appear, especially in developing countries; and providing open access to the full richness of historical and current mathematics literature.

In what follows we review the fundamental role of mathematics in modern life and society, and explain the IMU's role in strengthening mathematics worldwide. We show how the IMU is organized, and highlight the roles of both its Commission for Developing Countries (CDC) and the International Council for Mathematics Instruction (ICMI), as well as their linkages and collaboration in projects in the developing world. The IMU's championing of a World Digital Mathematics Library is discussed briefly.

This paper then concludes with a 4-page roadmap for future action.

1. The role of mathematics today

While mathematics may not be fully appreciated by people who are not mathematicians, it is omnipresent and essential to our modern technological society. The functions of mathematics guide many of the habits we have come to take for granted: talking on the cell phone, following friends on Facebook, searching by Google. If the language of mathematics is foreign to most people, we benefit from its importance every time as we use our automobiles, laptops, security systems, traffic models, medical scanners, and countless other features of modern culture.

Some areas of mathematical research are extremely specialized, and are home to only the few participants able to follow the latest discussions. On the other hand, enriching cross-currents also swirl through the mathematical universe, breaking down internal barriers and opening very interesting and mathematically compelling problems to general – and highly productive – discussion.

The design of microprocessor chips is done by mathematical methods, especially those of discrete mathematics. In industry, discrete optimization is revolutionizing how products are manufactured, ordered, stored, and delivered. The field of pattern theory has revolutionized developments in computer vision, speech recognition, signal processing and parts of artificial intelligence. The life sciences in particular are bursting with new mathematical power. Optimization techniques help predict protein folding and unfolding. Analytical tools unlock the value of complex data sets generated by genomics and proteomics. Knot theory, along with probability theory and combinatorics, helps biologists understand the three-dimensional mechanics of DNA sequencing. In silico modeling replaces expensive and dangerous experiments in medicine, aeronautics, and other fields. All told, mathematics is revolutionizing practices in health care, energy, agriculture, economics, public policy, political science, environmental studies, public transport, logistics, and other fields.

The contributions of mathematics to modern life extend beyond its partnerships with the other sciences and engineering. Over the last several decades it has become clear that a mathematically-educated population is a key to economic development. Before the 1950s, few people suspected the paramount importance of academic research to economic development, which was assumed to be almost entirely a product of capital and labor. However, as suggested by, e.g., the pioneering work of MIT economist Robert Solow, "technical progress," based on new knowledge, may well be more important than capital or labor in driving economic growth[1] . This discovery drew many more economists to look deeply into the contributions of knowledge to growth. While this exploration was proceeding, international development agencies continued

[1] Robert M. Solow, *A Contribution to the Theory of Economic Growth*, Quarterly Journal of Economics (MIT Press), 70 (1): p. 65 – 94, 1956.

to place emphasis on primary and then secondary education, largely neglecting tertiary education as a means of improving economic growth and mitigating poverty. From 1995 to 1999, the proportion of development aid allotted to higher education declined to just 7%. By the turn of the century, however, this began to change. In 1999, the World Bank published *Knowledge for Development*, an influential report that looked at how developing countries could use knowledge to narrow the income gap with rich world economies. It showed a correlation between education in mathematics, science, and engineering and improved economic performance.

Subsequent studies showed that not only primary and secondary but also tertiary education can raise GDP directly and, for developing countries, increase the speed of catch-up[2] . A study focused on the experience of Taiwan showed that higher education played a strong role in the country's economic growth, finding that a 1 per cent rise in higher education stock (as defined by those who had completed higher education, including junior college, college, university, or graduate school) led to a 0.35 per cent rise in industrial output[3] . This work examined the effects of concentration in different disciplines and concluded that study of the natural sciences (including mathematics) and engineering had the largest effect. Other studies have looked more closely at the output of STEM schooling and found that it leads specifically to higher cognitive skills, which in turn affect the economic growth rate. One analysis, examining mathematics and science testing from 1960 to 2000, showed that the level of cognitive skills of a nation's students has a large effect on its subsequent economic growth rate[4] . More specifically, the authors concluded that a highly skilled work force can raise economic growth by about two-thirds of a percentage point every year. The experience of Korea, host of the 2014 International Congress of Mathematicians, provides a typical illustration.

Another useful tool to improve education is the Mathematics Olympiad. In Brazil, for example, some 20 million students participate each year in the Brazilian National Mathematics Olympiad, organized by the Institute for Pure and Applied Mathematics in Rio de Janeiro. The country's scientific leaders see this exercise as a means of moving general education toward the levels of the developed countries of the North.

A related issue of importance is whether a country should invest in "just a few rocket scientists" at the very top of the ability distribution or in "education for all." The answer seems to be that both are separately important to economic growth. Substantial numbers of outstanding scientists, engineers, and other innovators are needed who can work at the frontier, but every nation also needs a labor force that has the basic mathematical skills required by technologically driven economies[5] .

Finally, while the strong link between cognitive skills and economic growth should encourage continued reform efforts, reformers should bear in mind that economic output alone is not sufficient.

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[2]Bloom, David, David Canning, and Kevin Chan, *Higher Education and Development in Africa*, Washington, DC: The World Bank, Human Development Sector, Africa Region, February 2006.

[3]T-C Lin, *The Role of higher education in economic development: an empirical study of Taiwan case.* Journal of Asian Economics 15 (2): 355– 371, 2004.

[4]Hanushek, Eric. A, Dean T. Jamison, Eliot A. Jamison, and Ludger Woessmann. *Education and economic growth: It's not just going to school but learning something while there that matters,* Palo Alto, CA: Hoover Institution Press, 2008. The authors state, "Our analysis suggests that increasing tertiary education may be important in promoting faster technological catch-up and improving a country's ability to maximize its economic output."

[5]Hanushek et al, ibid.

Addressing challenges for the present and the future

Strengthening the mathematical culture of a society can begin with several clear-cut objectives. For example, the mathematical community is somewhat isolated by the difficulty of its subject matter, so that the public poorly understands what mathematicians do. In many countries, the mathematics community could make a better case for the value of mathematics to the public and the government. Communicating this status is especially important because so few political leaders have mathematical or scientific backgrounds. Mathematicians who are not accustomed to explaining or marketing their subject have an opportunity to articulate the value of what they do to those outside the scientific community.

A related challenge is the perception that training in mathematics is uninteresting or unnecessarily arduous – a long journey into a "foreign" world that is accessible only to certain kinds of minds. Here, the responsibility lies primarily with the teachers, from whom the student can benefit from clarity and inspiration at every level. At any step of the mathematical journey, a keen teacher can inspire and excite the student's enthusiasm, and open a window to the unfolding power of a good mathematical education. School systems and countries hoping to encourage interest in mathematics can take care to support excellent programs at every level simultaneously, from primary education to research in universities. When well-qualified people are teaching effectively at each of these levels, students climb the educational ladder with excitement and see themselves as some day replacing them the teachers they admire. This journey is doubly arduous in settings where support for mathematics education is scant. For example, in most African countries, mathematical development is limited by low numbers of secondary school teachers and of mathematicians at the master's and PhD levels. Countries with too few professors to train the next generation of leaders face the challenge of building up their training capacity and developing up-to-date methods and systems. Faculty and graduate students who suffer from professional and geographic isolation should be encouraged to form partnerships and networks for sharing ideas with colleagues and generating the research they must do to advance professionally.

In the least developed countries (LDCs), students who arrive at the university hoping for education in mathematics or science may find especially daunting challenges, beginning with over-crowded learning spaces. Classrooms originally designed to hold 30 or 40 students may be jammed with several hundred youngsters, seated elbow to elbow, balancing on window sills, standing along walls. Teachers should also work hard to bring the university curriculum in sync with career realities. Curricula seldom include career guidance for the student, who simply finishes as a "math major." Aligning the curriculum with real-world job opportunities – not only in teaching, but perhaps in IT, finance, computing, or bioinformatics – allows students to see for themselves a potentially exciting future[6] . In most developing countries, teaching and research infrastructure is inadequate. Few buildings have adequate electrical wiring, let alone the internet access that is taken for granted in modern universities. Students seldom have access to textbooks or journals, and libraries built decades ago are not equipped to provide access to digital resources. The few functional public computers are kept in tiny labs, where students must share, peer over shoulders, or wait their turn; few students can afford their own. Inadequate bandwidth hinders Internet use and downloading even when one succeeds in getting online.

Themes of research often focus exclusively on traditional branches of pure mathematics such as

[6] This is the objective of the current International Commission for Mathematical Instruction (ICMI) Study 20, *Educational Interfaces between Mathematics and Industry (EIMI)*, see `http://www.iciam.org/EIMI/`

algebra, geometry and analysis. In some countries, curricula offer poor guidance in probability, statistics, and applied mathematics, emphasizing pure and compartmentalized mathematics by tradition. In contrast, other countries assign low priority to pure mathematics, endangering the long-term integrity of their programs.

Excessive teaching loads discourage both faculty and students. Students have too little attention; faculty have too little time for their students, and too few incentives, funding, or potential partners for themselves and their research. Seldom able to attend professional conferences, they have slight exposure to the people and ideas of their field.

At a more practical level, lecturers are poorly paid compared with their counterparts in government and the private sector, while still facing high costs of living. A teaching career at some universities may require a second job as a clerk, high school teacher, or taxi driver if one is to support a family. Nor do mathematics graduates have many options in the private sector, which has only recently begun to hire mathematicians. By comparison, university mathematics graduates in developed countries often have their choice of many appealing careers. In Germany, for example, a recent survey of average starting salaries for university graduates indicated that mathematics graduates were second-highest. Only 20 percent of them took teaching jobs, while the rest had a wide choice of options, many of them in industry.

Therefore, while mathematics is poised to continue expanding in scope and vitality, nourishing other scientific fields and being nourished by them in return, it has reached a significant point in its evolution. Many developed countries support excellent programs at their leading institutes and universities, often enriched by talented students from abroad, but the level of primary and secondary education is often inadequate to prepare the next generation of mathematicians. In developing countries, especially the least developed, raw talent is abundant, but almost completely untapped. As it becomes clearer each year how desperately the major challenges of disease, hunger, climate change, environmental remediation, and energy development depend on mathematical, computational, statistical and other quantitative skills, the urgent task of developing latent mathematical talent should be a high priority everywhere.

To address our most urgent economic, developmental, and societal challenges, we should hold up and adhere to a vision of mathematics as a living science, connected to the real world of people, institutions, and countries. Countries need more support for those who wish to become educators and researchers in mathematics, and they need more collaboration among institutions and people seeking to make this happen. Necessary steps include stronger teaching of primary and secondary students; more government support for teachers, faculty, and infrastructure; scholarships for graduate students and fellowships for faculty; and a clearer delineation of pathways to rewarding mathematics-based careers.

The International Mathematical Union can play a larger role in helping this come about, and has already taken preliminary steps indicating its determination to do so. The World Mathematical Year 2000, for example, which was initiated by the IMU, resulted in educational and communications activities around the world, reaching many people who had no previous exposure to the breadth and relevance of mathematics. At a national level, the Mathematical Year 2008 in Germany enhanced the visibility of mathematics through thousands of educational activities for all levels, events for the general public, topical workshops, and meetings with industry leaders and journalists. Recently, the Mathematics of Planet Earth 2013 sponsored events world-wide and invited participants to contribute to a daily blog recording the activities of mathematicians who study the evolving processes of planet earth.

2. Strengthening mathematics world-wide: IMU's role

The International Mathematical Union (IMU) is a global organization that addresses mathematics as an international activity, brings together mathematicians from all disciplines, and recognizes the challenges described above. Its goal at present is to strengthen and expand its ways of dealing with those challenges. It is well positioned to do so, partly because the members of IMU are countries – not individuals – and national governments are uniquely positioned to develop country-wide policies and scale up successful programs for the common good. The primary and founding purpose of the IMU, according to the organization's statues, is simply to "promote international cooperation among mathematicians." The emphasis on the word "cooperation" is deliberate, because it reflects the collaborative nature of mathematics, as discussed in the previous section, and suggests that mathematics provides a common language for scientists and engineers the world over. And it is a language that spans the spectrum of mathematical activities, from the "blue-sky" ruminations of pure mathematics to the practical, interdisciplinary pursuits of the applied mathematician interacting with other scientists and engineers.

While mathematicians for the most part carry on their business without any thought for the IMU, it has been since its founding the principal body representing mathematics on the world stage[7] . In terms of structure, the IMU is an international non-governmental, non-profit scientific organization. It is a member of the International Council for Science (ICSU), a Paris-based umbrella organization. The IMU is governed by a set of rules of seeming simplicity that have nonetheless been debated with great vigor over the decades. Its full menu of objectives is:

- To promote international cooperation in mathematics;
- To support and assist the International Congress of Mathematicians (ICM) and other international scientific meetings;
- To encourage and support other international mathematical activities considered likely to contribute to the development of mathematical science in any of its aspects – pure, applied, or educational[8] . The IMU supports the whole range of those engaged in the study or practice of mathematics, including those of limited mathematical experience, beginners contemplating a major in the subject, science students who aim toward careers in science and technology, and researchers working at the pinnacle of their subject. Beyond supporting the profession itself, the IMU feels strongly that all educated people should attain a degree of mathematical literacy if they are to be informed citizens in our knowledge-based era.

While raising the level of mathematical literacy world-wide is desirable, however, the IMU for most of its history has never had the resources to even begin to think at a global scale. In fact,

[7]For a full description of the IMU from its founding through the early 1990s, see Ollie Lehto's detailed and evenhanded account, *Mathematics Without Borders: A History of the International Mathematical Union* (Springer, 1998). Of special value is the appendix, which includes chronological lists of country membership, medal winners, leaders, and meeting sites of not only the IMU but also its several committees and commissions. See also *Mathematicians of the World, Unite! International Congress of Mathematicians – A Human Endeavor*, Guillermo P. Curbera, Taylor & Francis Group , 2009.

[8]These simple objectives, proposed in 1949 by American mathematician Marshall Stone, have remained virtually unchanged. As leader of the effort to re-start the Union, Stone pressed hard for inclusiveness, carefully navigating both post-War and Cold War sensitivities. A "Declaration of Universality" helped set the tone for international cooperation that has endured many trials in the ensuing decades.

the IMU has been known to most mathematicians for just two functions:
(1) organizing the quadrennial International Congress of Mathematicians (ICM), a showcase of current research, and
(2) awarding the Fields Medal, widely regarded as the mathematical equivalent of the Nobel Prize. The medals are named after J. C. Fields, a leading Canadian mathematician who worked tirelessly for international collaboration in mathematics and willed part of his estate to endow the medal[9] .

Structure and function

The structure and function of the IMU have been considered with great care from the outset, and are changed only after broad and slow consultation with the greater mathematical community. The International Mathematical Union has no personal members. Its members and associate members are countries [10] represented through an "Adhering Organization," which may be its principal academy, a mathematical society or other mathematical institution. A country starting to develop its mathematical culture and interested in building links to mathematicians all over the world is invited to join IMU as an associate member. Multi-national mathematical societies and professional societies can join IMU as affiliate members. In mid-2014, the IMU has 71 members, 10 associate members, and 4 affiliate members.

Every four years the IMU membership gathers in a meeting of the General Assembly (GA) that consists of delegates appointed by the Adhering Organizations, together with the members of the Executive Committee (EC) and other invited guests. Most formal decisions are made at the GA, including the election of the officers, establishment of commissions, the approval of the budget, and changes of the statutes and by-laws. The EC consists of the President, two Vice-Presidents, the Secretary, six Members-at- Large, all elected for a term of four years, and the Past President. The EC is responsible for all policy matters and for such tasks as choosing the members of the ICM Program Committee and various prize committees. The EC typically meets once a year physically, though most of its business is carried out by e-mail. The 10 members of the Executive Committee (EC), who serve for four years, are elected by the General Assembly from a slate of candidates proposed by the Nominating Committee after consulting the IMU members. One of the major jobs of the EC is the preparation of the next Congress. This begins with site selection, and recommendation of that site to the General Assembly for a vote. The next step is appointment of the ICM Program Committee, which plans the program, decides on the total number of lectures and specialized panels, and assigns to each panel a number of lecture slots. The last four ICMs were held in Hyderabad (2010), Madrid (2006), Beijing (2002), and Berlin (1998). The venue for ICM 2014 is Seoul.

Members are admitted into one of five groups, depending on the stage of their mathematical development and contributions, such as publications. Group number determines the number

[9]J.C. Fields proposed awarding two to four gold medals every four years to honor outstanding mathematicians, a suggestion accepted at the 1932 ICM in Zurich. He urged that the award be considered both in recognition of work already done and as "an encouragement for further achievement." This was much like the original intent of the Nobel Prize, which has since come be regarded as a reward for past achievement, while the Fields is considered a prize for young mathematicians, and since 1966, those "not over forty years old." The medal itself, of 14-karat gold, was designed by Canadian sculptor Robert Tait McKenzie to represent Archimedes. Fields was preparing his proposal for the Zurich meeting when he fell ill. He died in August 1932, one month before the Congress. His proposal was accepted at the closing session, as was his own posthumous cash contribution toward a cash prize to accompany the medal. For a picture of the Fields medal (and other IMU award medals), see `http://www.mathunion.org/general/prizes`.

[10]See e.g. `http://www.mathunion.org/members/countries/list/sorted-by-continents` for a list.

of votes and the level of dues, which have always been paid in Swiss francs; now that the IMU has a permanent Secretariat in Berlin, Germany, a change-over to Euros will be proposed at the 2014 GA. Dues are set in terms of units; for members of Groups I, II, and III, the number of unit contributions were originally set as one, two, and three, respectively. For Group IV they were five, and for Group V eight (these have since been modified to increase the dues progressively). Until recently, these dues provided virtually all of the IMU's budget, except for small contributions from international organizations. A major topic today, however, is increasing the budget so the IMU can take a more active role in supporting mathematics in developing countries and promoting a global Digital Mathematics Library.

To preserve its history, the International Mathematical Union maintains an archive containing important correspondence and documents. The archive was first located in Zurich and was moved to the University of Helsinki in 1994. In 2011 the archive was moved to the new permanent IMU Secretariat in Berlin and officially opened on November 10, 2011.

Some IMU traditions

A notable feature of the IMU has been its frugality. David Mumford, president in 1994, said he was told by French mathematician Jean-Pierre Serre the two secrets of the Union's success: "First, no one was ever nominated to the Executive Committee who wanted the job; second, the IMU has no money to speak of." For the years 1952-1954, the total budget was $3,965, to be used for secretarial help, office expenses, traveling expenses of the Executive Committee, and emergency and reserve. As Prof. Lehto writes, only slightly tongue-in-cheek, "All mathematical plans and activities were largely imaginary, because the Union had practically no funds of its own for their implementation." In fact, the IMU has been able to accomplish a great deal through its frugality and through the tireless volunteer activity of members who are willing to serve on committees, organize programs, and coordinate membership. The budget for this work was expected to be covered entirely by members' contributions. Dues are considerably higher today, though still tiny compared with other scientific unions or the assets of private foundations, for example. Dues for 2012 were 1,670 Swiss Francs per "unit," the quantity by which all dues are calculated. By this schedule, a Group I country, such as Cameroon, pays 1,670 Swiss francs per year, or one unit, while a Group V country, such as the United States, pays 20,040 Swiss francs, or about 12 units. Dues have increased a modest 2% per year in the last few years. The budget for 2012 totaled 536,000 Swiss francs, and major activities (from a financial point of view) were scientific activities of the Commission for Developing Countries, CHF 120,000[11] ; travel grants, CHF 62,000; and ICMI scientific activities, CHF 40,000.

Another tradition that has served the IMU well is openness. One aspect of openness is to welcome other organizations and disciplines to meetings of the General Assembly, as it did in the 1950s with the International Union of Theoretical and Applied Mechanics, for example. Since the late 1970s, its presidents have all spoken in favor of increased collaboration with organizations representing computer science, mathematical physics, history, probability, and other fields. In his 1986 presidential address, Lennart Carleson said: "In my opinion, it is essential for the health and vitality of our science to extend and cultivate our relations to other fields of applications."

With regard to partnerships, the Union has developed overlapping policies that seem to co-exist peacefully. On the one hand, it has taken care to avoid binding commitments that might limit its freedom or impose obligations. On the other hand, it has greatly enriched its programs

[11] This includes a generous grant of US $45,000 from the Abel Fund of Norway.

through a variety of historical and current collaborations. Some of these date to its earliest years, such as a close relationship with both UNESCO and ICSU, the International Council of Scientific Unions (now called simply the International Council of Science). Both organizations have provided small but essential annual grants. ICSU has been an important ally in the commitment of both organizations to the "free circulation of scientists," which has allowed IMU meetings to maintain their openness even during the Cold War and the era of apartheid in South Africa. UNESCO has been a partner in many programs, including the initiation of the Capacity and Networking Project (CANP).

In addition, the IMU has benefited from small and large partnerships with many organizations. The Volunteer Lecturer Program has been supported by the U.S. National Committee on Mathematics and the French International Center for Pure and Applied Mathematics (CIMPA). The MARM program, Mentoring African Research in Mathematics (MARM), has been supported by the Nuffield Foundation and Leverhulme Trust in Great Britain. The African Mathematics Millennium Science Initiative (AMSSI) has been supported by the IMU and the Mellon Foundation, and now the Simons Foundation supports the Africa Mathematics Project (AMP). These partnerships and many more allow the IMU to attempt its world-wide reach – and bring hope that this reach can be extended still further.

Another aspect of openness is the inclusion of both pure and applied mathematics. In principle, the IMU works hard for a balance between the two broad areas, and indeed, many revered mathematicians, including Archimedes, Newton, Gauss and von Neumann, regarded mathematics as a seamless whole. And many of today's applied mathematical tools – the calculus, Fourier series, and matrices, with their powerful economic and scientific impact – were once considered "pure" research. Nonetheless, the two ends of the spectrum have often experienced a mutual distance, and the International Congress of Industrial and Applied Mathematics and the International Council of Industrial and Applied Mathematics were formed in part because some applied mathematicians felt ignored by the IMU. Similarly, mathematicians and mathematics educators have often struggled to work in partnership, despite the fact that the IMU embraces both activities within the same informal charter. Both groups today profess a strong desire to work in partnership, and indeed many of the most vital IMU activities, especially in the developing world, are driven jointly by leaders in both the IMU and ICMI.

A central premise of the IMU is that mathematics is a collaborative endeavor and it is enriched to the degree that people from all regions are free to interact. "Promoting international cooperation in mathematics", one of IMU's primary goals, has proven complex in a world of political tensions. Ollie Lehto's book *Mathematics Without Borders: A History of the International Mathematical Union* (Springer, 1998) chronicles its struggles to remain objective and apolitical, a tenet that is still central to the IMU today.

Medals of recognition

The IMU recognizes the work of outstanding mathematicians through prizes – again, many of them supported by partner organizations. The value of prizes has been debated over the years; some mathematicians have criticized them as more extravagant than useful. Educators have expressed concern that they encourage elitism. But the consensus is that prizes serve the valuable functions of recognizing models and setting norms, and their presentation begins each ICM. The number of medals and other distinctions awarded has increased over the years, in an effort to reward achievement the newer interdisciplinary areas as well as established sub-fields.

• **The Fields Medal** (1936) recognizes outstanding mathematical achievement for existing work and for the promise of future achievement; awarded to mathematicians not exceeding the age of 40. The cash award for this prize was endowed by a bequest from the Canadian mathematician J.C. Fields.

• **The Rolf Nevanlinna Prize** (1986) is awarded for outstanding contributions in the mathematical aspects of information sciences, to a researcher not exceeding the age of 40; the IMU considers it the equivalent of the Fields Medal for this very mathematical field. This prize is financed by the University of Helsinki.

• **The Carl Friedrich Gauss Prize** (2006) honors a scientist whose mathematical contributions have found significant application outside mathematics. This prize is sponsored by the German Mathematical Society.

• **The Chern Medal** (2010) is awarded to an individual whose lifelong achievements in the field of mathematics warrant the highest level of recognition. This prize is financed by the Chern Medal Foundation and the Simons Foundation.

• **The Leelavati Prize** (2010) accords high recognition and great appreciation for increasing public awareness of mathematics as an intellectual discipline and the crucial role it plays in diverse human endeavors. From 2014 onwards, this prize is sponsored by Infosys.

• **The ICM Emmy Noether Lecture** (1994) is a special lecture which honors women who have made fundamental and sustained contributions to the mathematical sciences.

• **The Ramanujan Prize** (2005) recognizes an outstanding mathematician from a developing country who is less than 45 years of age. Up to 2012 it was supported financially by the Norwegian Abel Memorial Fund; from 2014 onwards it is supported by the Department of Science and Technology (DST) of the Indian government. It is jointly awarded by IMU, the International Centre for Theoretical Physics, Trieste, and the DST.

IMU commissions and committees

Much of the work of the IMU is divided among several commissions and less formal committees. The standing committees focus on the following topics:

Mathematical Education: IMU's research on mathematics education is done through its **International Commission on Mathematical Instruction (ICMI)** which was established in 1908, before IMU was founded. This commission is organized like the IMU through its own Executive Committee and General Assembly.

Developing Countries: A significant percentage of IMU's budget, including grants received from individuals, mathematical societies, foundations, and funding agencies, is spent on activities for developing countries. The IMU took its first organized steps towards the promotion of mathematics in developing countries in 1971. Since 2011 this has been done through the **Commission for Developing Countries (CDC)**.

History of Mathematics: The **International Commission for the History of Mathematics** (ICHM), created in 1971, is operated jointly by the IMU and the Division of the History of Science (DHS) of the International Union for the History and Philosophy of Science (IUHPS).

Information and Communication: The **Committee on Electronic Information and Communication (CEIC)** was formed in 1998 and advises IMU on matters concerning mathematical information, communication, and publishing. A special current focus is its effort to help develop a world-wide Digital Mathematics Library.

3. The IMU and the developing world

In the scientifically developed world, a career in mathematics has long been respectable, if not particularly remunerative. In the developing world, however, the prospective mathematics student faces many barriers and less remuneration. There is little popular interest in mathematics, little support from friends or family, few role models among teachers and even university faculty, and scant financial aid. It may be rare to find public funding for MS or PhD studies, as is the norm in many advanced countries. Talented students, especially in Africa, often leave home to find institutions that offer scholarships or research groups, contributing to brain drain.

Weakness is found in Latin America, Asia, and Africa at every level of development. As underlined in the recent report *Mathematics in Africa: Challenges and Opportunities*, "To concentrate on primary education alone will be futile if there are no qualified teachers; there can be no qualified teachers without skilled mentors to train them." [12] A healthy educational systems produces sufficient numbers of students at higher secondary and tertiary levels to replace retiring faculty and share the heavy teaching load. Supporting this young population supposes a societal recognition of the importance of the profession and improved working conditions of teachers. It requires system-wide efforts to allow teachers access to networks, resources, and in-service development.

The difficulties do not end for individuals who complete their studies. Faculty members in many developing countries experience both professional and geographic isolation. With overwhelming teaching loads, they have little opportunity to do research, and few or no graduate students with whom to form a critical mass of investigators. Government support is meager and faculty tend to absorb whatever financial aid is available simply to support themselves.

It does not appear that lack of advanced mathematical communities is caused by lack of indigenous talent. It seems far more likely that mathematical talent is distributed randomly and is not favored by geographical or political boundaries. But the opportunities to develop talent vary widely depending on circumstances. For example, in former British and French colonies in Africa, such as Kenya and Senegal, and in Vietnam, there are excellent pre-university schools that graduate excellent students in mathematics. But such opportunity is available only to a small percentage of young people, while the vast majority are held back by lack of opportunity caused by poverty or other conditions. A key responsibility of the IMU is to take what action it can to support those regions of the world that lack mathematical resources, infrastructure, or expertise.

Developing an outreach mission

The IMU first turned its attention to the needs of developing countries in the 1960s. With the end of the colonial era, newly independent countries wanted to build their own structures of research and education, but few of them had the needed resources. Unfortunately, the IMU, with few resources of its own, could offer little help.

The IMU took its first organized steps to promote mathematics in the developing world in the early 1970s, when it formed an International Group in 1971 to advise the Executive Committee. In 1974 it helped organize Regional Groups, and in 1975 its Commission on Exchanges began to devote its energies to developing countries. An important step of this Commission was made in Africa, in 1976 at Rabat, Morocco, when it used a grant from the Canadian International

[12] Produced by the Developing Countries Strategic Group (DCSG) of the IMU for the John Templeton Foundation, 2009.

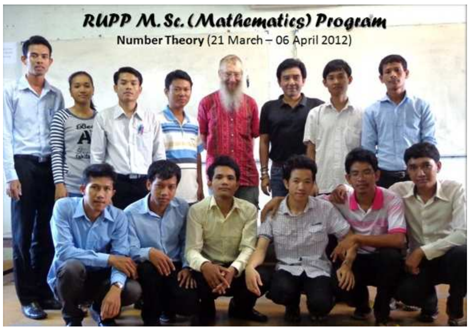
Development Agency (CIDA) to support the All African Mathematical Conference in Morocco, which led to the creation of the African Mathematical Union (AMU). In 1978 the IMU took another important step in creating a Commission on Development and Exchange (CDE), whose aim was to support mathematicians based in economically disadvantaged countries through individual research grants and travel grants to conferences.

In the late 1980s, the CDE was able to extend its work thanks to contributions from national mathematical societies. The CDE built links to the South-East Asian Mathematical Society, the African Mathematical Union, and the Latin America School of Mathematics (ELAM), as well as Centre International des Mathématiques Pures et Appliquées (CIMPA) in Nice, France, and the International Center for Theoretical Physics (ICTP) in Trieste.

After over a decade of awarding small grants through the CDE, the IMU felt it had to do more. In 2002 the Executive Committee, under the leadership of then-President John Ball, convened an ad-hoc committee to advise on strategy. This committee reported in September 2003, advocating a new Developing Countries Strategy Group (DCSG), along with a new class of IMU associate membership for less mathematically developed countries. An annual grant from the Norwegian Abel Fund helped the CDE support mathematics in the developing world, and to award the Ramanujan Prize. Additional grants have come from funds administered through the London Mathematical Society and American Mathematical Society. The CDE also helped support the study "Mathematics in Africa: Challenges and Opportunities" (2009), funded by the John Templeton Foundation.

In order to accommodate its growing palette of such activities, the IMU decided to consolidate them under a new entity called the Commission for Developing Countries (CDC), which in 2008 became a major feature of the IMU's mission. It now supports a number of new projects, including:

• **The Volunteer Lecturer Program (VLP)**
The VLP, with help from CIMPA of France and the U.S. National Committee on Mathematics, sends lecturers to give intensive 3- to 4-week courses in mathematics at the advanced undergraduate or master's level. Since 2007, courses have been given in Africa, Central America, South East Asia and the Middle East on topics in core mathematics. The volunteer lecturer is chosen by the host institution from an online database maintained by CDC. The volunteer lecturer's home institution is asked to provide leave with pay. The hosting university has no financial obligations, but is expected to provide local assistance.

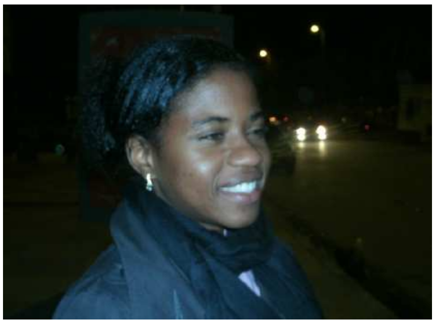


A VLP group in 2012.
(Photo by Fidel Nemenzo)

• **Mentoring African Research in Mathematics (MARM)**
IMU and AMMSI assisted the London Mathematical Society in founding the MARM program, which supports mathematics and its teaching in the countries of sub-Saharan Africa. Through a mentoring partnership between mathematicians in Africa and those in the United Kingdom and Europe, it cultivates longer-term relationships between individual mathematicians and students. The program is designed to counter brain drain by supporting qualified mathematics professionals in situ. The program is financially supported by the Nuffield Foundation and the

Leverhulme Trust.

• **The African Mathematics Millennium Science Initiative (AMMSI)**



AMMSI administers a scholarship program for mathematics graduate students on the African continent and helps to organize workshops, conferences, and lectures. It was initiated by the Mellon Foundation and the Science Initiative Group, based in Princeton, NJ. Since the end of Mellon funding in 2010, it has continued to receive a modest grant from CDC to sustain the scholarship program. AMMSI is structured as a distributed network with regional offices in Botswana, Cameroon, Kenya, Nigeria and Senegal. Applications can be sent to the AMMSI Secretariat `ammsi@uonbi.ac.ke`.

Fatou Néné: AMMSI scholarship in 2005; Ph.D. in 2010; now Lecturer at AIMS-Senegal, Dakar. (Photo by Wandera Ogana)

• **Mathematics Library Assistance Scheme for Developing Countries**
The CDC offers to pay the shipment costs when individual scientists or institutions wish to donate books in the mathematical sciences to libraries in developing countries. Interested universities or research institutions can send a list of mathematical areas in which books are needed to the CDC Administration. More information can be found at URL
`www.mathunion.org/cdc/further-cdc-activities/library-assistance-scheme`.

• **Adopt a Graduate Student**
The largest single need among mathematics communities in developing countries is graduate student support. It is common for students in developed countries to find public or other sources of support, but this is not the case in the developing world. There are no teaching assistants, because the faculty themselves teach even the most basic courses simply to earn enough money to support themselves. Graduate students must often take a job (perhaps teaching in a remote village) to subsidize their studies. A grants program provides a high return on investment in countries where a year's support typically costs the equivalent of US $1000-3000.
The Adopt-a-Graduate-Student program, designed to address this need, will be launched at the 2014 ICM. The pilot design, initiated by the Friends of IMU, aims to match interested donors, especially from the mathematics community, one-on-one with talented mathematics graduate students in need of a graduate assistantship to continue his/her studies at a university in the developing world.

Support from the CDC made it possible for these graduate students (four from Cambodia, one from Laos) to continue their Mathematics Master Program in the Philippines. (Photo by Fidel Nemenzo)

The Adopt-a-Graduate-Student program, and other IMU fundraising efforts to benefit the CDC,b strive to make more such graduate fellowships possible for students from and in developing countries.

While the IMU's activities in the developing world are limited by its small budget, it collaborates with many other initiatives, such as the following:

• A material influence on the CDC's work in recent years has been a two-year MSc program in mathematical modeling run at the University of Zimbabwe in the late 1990s and at the

University of Botswana subsequently. It was funded primarily by the Norwegian Program for Development, Research and Education (NUFU). While the IMU was able to add only small amounts of additional conference support, it has benefited substantially from the pioneering work of the NUFU program. Between 1997 and 2007, the modeling program was able to produce some 50 MSc graduates and 10 PhD graduates, many of whom successfully moved to faculty positions in southern Africa. The program moved to Botswana and came to an end after a decade. It was replaced by a new program called NOMA, based at the University of Dar es Salaam, Tanzania, but it continues to serve as a valuable model. Among its chief advantages was flexibility, which allowed resources to be used either for travel for staff to teach in Zimbabwe, student travel to the host institution, or student travel to professional conferences. The program also improved cooperation among mathematicians of the region and established links with industry.

• **Africa Mathematics Project**

This program is funded entirely by the Simons Foundation, but the IMU consulted on its design, and provided context and rationale through its earlier report Mathematics in Africa: Challenges and Opportunities. The Simons project, which began in fall 2012, will focus on mathematicians and their graduate students at institutions of higher learning in sub-Saharan Africa. The Foundation will make competitive awards that, taken together, will total approximately 400,000 USD per year for each of the next 10 years.

Mathematics in Emerging Nations: Achievements and Opportunities (MENAO)

The IMU is organizing a major one-day satellite event for the Seoul Congress in 2014 to highlight mathematical activities and young talent in developing countries. The MENAO event will take place on August 12, 2014, the day before the medal award ceremony, and is expected to attract mathematicians, potential donors or sponsors, governmental and NGO leaders, and representatives of industry. The South Korean Organizing Committee has generously raised the funds to invite 1,000 mathematicians from emerging nations to attend ICM 2014. This freed up CDC funds earmarked for the support of ICM participation of mathematicians from developing countries, part of which supports the organization of the symposium.

MENAO itself is both a gesture of support for mathematics in the developing world, and a reminder of Korea's own acceleration to scientifically-advanced status.

4. The ICMI and the challenge of mathematical education

The IMU has not only the challenge of supporting mathematics research in every nation, but also the twin challenge of strengthening mathematics education. In fact, its concern with education extends back as far in time as its research concerns, given that ICMI, was actually founded a decade before the IMU, in 1908. The affiliation of researchers and educators is natural enough, since researchers count on well-prepared students to form their graduate groups, and math educators should be able to do the preparing. Likewise, researchers are almost always educators as well, and so have a natural interest in the teaching process.

Worldwide, however, the challenges faced by mathematics education in society are probably more complex than those facing mathematics research. In scientifically advanced and in developing countries alike, students fail to meet current expectations, both in terms of advanced mathematics and in the basic mathematical literacy required for active citizens in technology-based societies. Too few teachers have had good mathematical preparation, and they lack adequate mathematical knowledge for teaching. For example, according to a recent report by the U.S. National Academy of Sciences, roughly 75 percent of U.S. students are not proficient in mathematics at completion of the eighth grade[13] , and no one has proposed an "easy" way to correct these deficiencies.

Barriers to mathematical literacy

In many societies, basic knowledge of numbers and measurements has been considered a sufficient basis for participation in society. But this is no longer sufficient in a technology-based culture. Too many school systems around the world emphasize memorization of rules whose meaning is not evident to students, introduce subjects without explaining what needs they meet, and offer students little freedom in doing their work. The modern world requires genuine mathematical literacy that includes the ability understand, analyze, and critically assess multiple data using various and complex systems of representation. It requires students who are able to make reasonable choices about uncertainty and risk. In short, it requires updating the objectives beyond standard arithmetic operations[14] .

While these difficulties pertain to countries around the world, weakness in mathematics is especially prevalent in developing countries. Few mathematics teachers have received adequate mathematical training. Young people avoid the subject for its perceived difficulty and poor career prospects. At advanced levels, students who find little or no public support leave academia for well-paying jobs in information technology, banking, or accounting. With an insufficient inflow of young faculty, senior professors retire without replacement. As the base of mathematicians shrinks, so does the chance of an enriching experience for students who remain. In an African university, for example, it is not uncommon for an academic department of mathematics to have only one or a few professors and two or three graduate students, making a collegial, productive work environment impossible.

In many countries, few women are encouraged to enter mathematics[15] . Yet while younger women lack sufficient encouragement and role models, they have convincingly shown an aptitude for science and mathematics comparable to that of younger men. For example, results of international calculations (OECD, 2004, 2007) tend to show that among the best-performing education systems are those with a gender-inclusive education within basic schooling.

To help meet the major challenges of humanity – in health care, environment, energy and development – substantial improvement is needed in mathematics education. This includes sustained political action in support of teaching and education. Without grass roots pressure from the bottom, and enlightened leadership from the top, too few children will have access even to basic schooling. Millennium Development Goal 3 – access to basic education for all by 2015 – is far from being realized; an estimated 72 million children are still not enrolled in primary school, and they would not find qualified teachers if they did enroll.

[13] National Research Council, *Successful K-12 STEM Education: Identifying Effective Approaches in Science, Technology, Engineering, and Mathematics* (2011), Washington, DC: The National Academies Press, 2011.

[14] This theme and others in this section were expressed by Michèle Artigue in her report *Challenges in Basic Mathematics Education*, United Nations Educational, Scientific and Cultural Organization (UNESCO), 2012. Dr. Artigue is past president of the ICMI, whose Executive Committee has affirmed this document as a position statement on mathematics education.

[15] On gender issues, see the important list of references on the website of the International Organization of Women and Mathematics Education (IOWME) at `http://extra.shu.ac.uk/iowme/`).

Another hindrance to a vibrant mathematics education is its poor public image. Many perceive mathematics as a fundamentally solitary activity, detached from interaction with others; as an arduous, silent and joyless effort devoid of any intuition; as an exclusive club, accessible only to a very small percentage of people. If this image was ever correct, it is outdated. Mathematics is a living science, the practitioners of which are eager to learn from each other, and excited about new developments. Becoming a professional mathematician does require arduous training, true, but the same is true for a professional athlete – a fact that has never prevented the multitude of others from enjoying sports.

Practicing mathematicians can do much to open students' eyes to the adventure of mathematical accomplishments through many centuries, on all continents. The ICMI and its members seek to convey this excitement as mathematics leads us deeper into advanced skills and into applications, including robotics, simulation, visualization, and modeling. Mathematics today extends far beyond the classroom as a living science, both anchored in the real world and interacting with other fields, notably computer science, biology, physics, astronomy, engineering, economics, and finance. Mathematics also builds the framework necessary for the abstract thinking and inventiveness that underpins many of the new approaches in these fields.

Perhaps the greatest general challenge to mathematics educators is to extend mathematics education to all – without lowering quality. As noted above, this is especially problematic in developing countries, which seldom have sufficient numbers of qualified teachers or sufficient physical resources to enrich the teaching environment. But the quality of teaching is far from satisfactory even in countries considered scientifically developed. Too many mathematics teachers are really "general-purpose" teachers without a degree in mathematics or expertise in mathematics training.

The ICMI understands that this is no longer acceptable. Mathematics itself has moved far and fast over the past few decades, and teachers should be prepared to convey this to their students. Teachers should be better equipped to present a vision of mathematics as a living science that interacts with many other fields. Even more important, they should also have knowledge beyond curriculum content – cognitive skills that allow them to understand the students themselves and how they learn[16] .

Since 1987, a part of the IMU's budget is allocated for the free use of ICMI, and ICMI and other branches of IMU collaborate on many levels. Most apparent is their joint emphasis on programs for the developing world, which have evolved in parallel and are often shared outright. Less apparent, perhaps, but equally important is the low-key approach of both groups. The ICMI, like IMU, has always seen its role as facilitator, rather than arbiter or rule-setter.

Bent Christiansen, a long-time vice-president of ICMI, articulated this position in 1982, when the ICMI was gaining focus: "ICMI should not be seen as powerful leaders of the development in mathematics education. In fact, the Commission and its EC should not decide what are proper or relevant solutions to problems in our field. But there was urgent need for a structure under which interaction and exchange of views can be facilitated."

ICMI and the issue of reform

Teaching, learning, and assessment are complex activities, spanning many domains. There are no "big theorems" of education, but instead a growing understanding of what the problems

[16]See below for a description of the *Klein Project*, which is designed to stimulate the interest and involvement of mathematics teachers.

are and how to begin to address them. In many countries, mathematicians have a tradition of involvement with pre-college education and teacher training, but it often concerns detecting and nurturing future mathematical talent. This is understandable, but does not reach the objective of quality education for all.

The second challenge is to achieve better collaboration between the various communities in charge of educational issues, in particular the mathematicians, teachers, and mathematics educators. In the last decades, the development of didactics as an academic research field has modified the traditional balance. Efforts to bring mathematicians into conversation with researchers in mathematics education have deeply influenced the IMU and initiated substantive change. Both the research and educational communities reach out to each other again: ICM has an educational component, creating opportunities for mathematicians to be aware of research in mathematics education; likewise, the ICME includes talks by mathematicians eager to share current research ideas with educators.

Many mathematics education researchers insist that mathematics for teaching should be seen as a form of applied mathematics whose knowledge differs fundamentally from a university mathematics education. Researchers explore didactical knowledge at the interface of the education and research. More broadly, research in mathematical education is a mixed science that joins mathematics (a natural science) with social sciences, including sociology, psychology, and neuroscience.

Alan Schoenfeld of the University of California at Berkeley, the winner of the 2012 Felix Klein[17] Medal, emphasizes the youth of mathematics education, which became a professional discipline only in the late 1960s and early 1970s. The mind of the student was regarded largely as a "black box," without much understanding of what was happening inside. As more cognitive knowledge was added to the field, researchers began to ask questions of more value, and learned to extract more meaning. "From 1975 to about 1990 we were building tools," said Schoenfeld. "From about 1995 to now things have grown more complex, and interesting." At the AMS annual meetings, he noted, mathematicians began talking about education. Now, about 40 percent of its sessions concern education.

Michèle Artigue, in her UNESCO article, states that the challenges of a quality education for all cannot be met without developing new educational knowledge through research. During the past decade, research in math education has developed strongly. At first it focused on students: learning processes, modes of reasoning and proof, representations and languages by which we access mathematical objects, and the potential of new technologies. More recently the focus has broadened to teachers: their beliefs about teaching and learning mathematics, how their knowledge and ability develop, and the cultural dimensions of teaching and learning.

Still to be understood, however, are the effects of educational choices and how to scale up new knowledge. Better research on didactic action is needed, probably incorporating other understandings from the cognitive sciences as these are acquired. Quality mathematics education for all is an ambitious challenge whose success depends primarily on the capacity of countries to develop and retain a sufficient number of qualified teachers; teachers able to present mathematics as a science anchored in history and living in the present; and teaching that connects

[17] At its 10th quadrennial congress, ICME-10 in 2004, ICMI inaugurated two prestigious awards, the Felix Klein and the Hans Freudenthal Medals (named for, respectively, the 1st and 8th presidents of ICMI), to be awarded at each consecutive ICME. The Felix Klein Medal is awarded for lifelong achievement in mathematics education research.

mathematics with other disciplines, especially scientific fields[18] .

5. The ICMI and the developing world

A new focus

After a somewhat rocky relationship over the past several decades, the IMU and ICMI have strengthened their linkages and synchronized their missions. A substantial development is their mutual focus on the developing world. This interest actually dates back to the early 1960s, when many former colonies were becoming independent, and education was seen as a vital factor in their development into full nationhood. But the desire to help had to wait several decades both because the IMU lacked sufficient resources for far-reaching programs and Cold War-related antipathies restrained collaborative action. Today, however, both institutions are far more focused on supporting developing countries, and finding more effective ways to leverage their resources. "Why do we do this?" asked mathematics educator Mary Kay Stein at ICME-12."Because finding ways to help others live a fulfilling life is partly what it means to be human."

Improving on older models of reform

A desire to help developing countries is not in itself a guarantee that it can be done effectively. Luckily, the ICMI, along with many other organizations, by now has long experience in trying – albeit not *always* successfully – to strengthen fledgling programs from afar. And they have had powerful feedback from experienced leaders of mathematics in the developing world. For example, a cogent description of when and how to implement reforms is offered by Bienvenido F. Nebres of the Philippines, a founder of the Southeast Asian Mathematical Society (SEAMS) with long experience in the Asian mathematics community.

Typically, writes Dr. Nebres, curriculum reform promoted by international agencies, such as the World Bank, Asian Development Bank, and consultants from the West, is organized in four phases. First comes the introduction of a new teaching approach inspired by a theory from the West ("new math," "back to basics", problem-solving, constructivism, etc.). Second is the development of textbooks and resources based on these approaches. Third come pilot and small-scale studies in particular contexts, which are "always successful." And fourth is national implementation, accompanied by a program of teacher training, which is almost never successful.
The early phases take up a good deal of time, and as the funding timetable moves along, each successive agent has less time to adapt, until finally the teachers charged with implementing the reform in their classrooms are left with two or three weeks to learn and apply the model. They must change their habits abruptly, with minimal training. The results are almost always bad, and a few years later, a new curricular project is launched to remedy the situation.

Dr. Nebres compares this caricature of reform to the regular reform timetable in Japan, which unfolds every 10 to 12 years. This process gives great importance to an orderly, grass-roots process that begins with systematic collection of suggestions from teachers, followed by their analysis, synthesis and discussion. Out of these discussions, the school systems – not outside agencies – decide on needed developments. Rather than a revolutionary attempt, writes Dr. Nebres, "we begin with what we have and improve on it, rather than wipe it out and totally

[18] For particular difficulties in retaining qualified teachers in developed countries, see the U.S. initiative *Math for America*, `http://www.mathforamerica.org/home`.

replace it."[19]

The ICMI has already learned much from leaders such as Dr. Nebres in designing programs for the developing world. Key among them is to work with local cultures in terms they understand, from the bottom up. These principles have been incorporated into the work of the IMU's Commission on Developing Countries (CDC), established in early 2008. Several new programs, designed and implemented jointly by the CDC and ICMI, have been received with enthusiasm in the host countries.

The Capacity and Networking Project (CANP)

Both IMU and ICMI are eager to work with any country able to make productive use of their help. A major development toward this goal is the new Capacity and Networking Project (CANP), designed to be planned, managed, and sustained locally by the host country. The goal of CANP, which is supported also by UNESCO and the International Council for Industrial and Applied Mathematics (ICIAM), is nothing less than to enhance mathematics education at all levels in developing countries. We give here a brief overview of the first CANP projects; see `http://www.mathunion.org/icmi/activities/outreach-to-developing-countries/canp-project/` for more information, including reports.

The first CANP gathering was held in 2011 at the University of Bamako, in Mali, with the additional Francophone countries of Burkina-Faso, Ivory Coast, Niger, and Senegal. The event was planned largely by a UNESCO representative working in Mali and Dr. Artigue of ICMI and CIMPA, in Nice, France. A UNESCO office in Bamako was able to assist in coordination and support.

The program, and in particular the workshop, built on existing activities in the region without seeking to reproduce or compete with existing development programs. The core of the program was a two-week workshop of about 40 participants, half from the host country and half from regional neighbors. It was primarily aimed at mathematics teacher educators, but also included mathematicians, researchers, policy-makers, and key teachers. The project also held associated activities such as public lectures, satellite workshops for students, and exhibitions for the general public.

(Photo: courtesy of Michèle Artigue)

After one year, a follow-up meeting in neighboring Senegal was held to ensure continuity and evaluation. A post-workshop and a one-year report were produced [20] , and this early program was used as a model for succeeding ones. The workshop was attended by a core group from each country to "establish that the spirit was still alive," said Dr. Artigue. "Before this event, the math communities in the countries did not work together. Now they are trying to do that. When you have few human resources, collaboration can make a difference. Education is not just teachers and learners. It is a function of a very complex system. You can't touch just one part. You can really achieve something if you touch the right people."

The second CANP was held in Costa Rica in August 2012, following the Mali model. Like

[19] Nebres, B.F. *Philippine Perspective on the ICMI Comparative Study*, in M. Menghini, F. Furinghetti, L. Giacardi, F. Arzarello (eds.)*Perspective on the ICMI Comparative Study: The First Century of the International Commission on Mathematics Instruction* Rome (1908-2008): Instituto della enciclopedia Italiana, 2009. p.281.

[20] See `http://www.mathunion.org/icmi/activities/outreach-to-developing-countries/canp-project-2011-2012-sub-saharan-africa/` . Reports on subsequent CANPs can also be found on the ICMI website.

most developing regions, Central America has little activity in mathematics education research. Costa Rica is the only country in the region to have a research group, which has been at the University of Costa Rica for two decades. This CANP invited 67 participants from Costa Rica, Panama, Dominican Republic, Mexico, Cuba, Spain, Venezuela, and Columbia.

"We included contemporary mathematics, including applied mathematics, fundamental math, the use of technology, and education research," said Angel Ruiz, Professor of Mathematics at the University of Costa Rica and ICMI member. "CANP is also a tool for in-service and pre-service teachers, and we try to establish links to primary education as well."

Meeting of the regional network formed as a result of the second CANP. (Photo: courtesy of Angel Ruiz)

Dr. Ruiz said that the key to improved education is better research. "How can you introduce the use of technology into the national curriculum?" he asked. "You need research backing you. That's why the Minister of Education came to us for this program. We think there is a connection between attitudes, or perseverance, and beliefs. After five minutes of working with a problem our local students give up. They lack perseverance. In Japan, the students try harder; they keep working. This is an example of a cultural habit we want to instill."

An important outcome of CANP 2012 was the creation of the Mathematics Education Network for Central America and the Caribbean, which organized a regional conference in the Dominican Republic in November 2013.

The third CANP was held in Cambodia, in October 2013; it included also representatives from Laos, Myanmar, Thailand and Vietnam. Most attendees were educators who prepare secondary mathematics teachers; a few mathematicians, teachers, and government curriculum personnel were included as well. Cambodia faces special difficulties: as of the summer 2012, only four Cambodians held a PhD in mathematics. This tiny community has plans to produce MSc (and eventually PhD) graduates and has built a relationship with IMU on the basis of which it is already producing MSc graduates in mathematics – indeed the IMU Volunteer Lecturer Program currently provides the faculty for Cambodia's only MSc program in mathematics.

CANP-2013 in Cambodia. (Photo: courtesy of Bill Barton)

Plans are also underway to produce mathematics texts in the Khmer language. The nation's development is clearly stifled by its lack of mathematicians, and the government is supportive of the CANP initiative.

CANP-4 will take place in September 2014, in Tanzania, aiming to promote networking in East-Africa.

'Experiencing Mathematics': A traveling exhibit.

Another important model that grew out of the partnership between the IMU, UNESO, and CIMPA in Africa is the traveling exhibit "Experiencing Mathematics." The goal of this project was to develop an exhibit with simple, interactive manipulatives that can help people understand how mathematics relates to science, technology, and the real world. Preparation began with funding from UNESCO in 2000, and as Dr. Artigue, a partner in the design of the exhibit,

recalls, the objective was to show visitors that mathematics is:
(1) astonishing, interesting and useful;
(2) accessible, in its first steps, to everyone;
(3) present throughout our daily lives; and
(4) important for our culture, development, and enrichment.

With continuing support from the IMU, ICMI, and UNESCO the exhibit was presented first at ICME-10 in Copenhagen in 2004. In 2005 it moved to Uganda, in sub-Saharan Africa, and then to Cambodia, where it was adapted primarily for teacher training. An unanticipated success, the exhibit has been presented to some 800,000 people in 20 countries, and continues to travel the world.

The Klein Project

Members of the ICMI suggested the Klein Project in memory of Felix Klein's famous book *Elementary Mathematics from an Advanced Standpoint*. Begun a century after the book was published in 1909, the project is intended as a stimulus to help mathematics teachers make connections between the mathematics they teach and advanced mathematics research. "There is a huge gap between the math in secondary school and the math people teach in universities," said Bill Barton, president of the ICMI, at a seminar at ICME-12 in Korea. "We thought it would be interesting for a project that communicates with teachers about math in a way that rekindles their love for their subject, in the same way Klein did. That is, ideas about the math they deal with on a daily basis that would enthuse and excite them, so they might become more exciting teachers." The project uses "vignettes," or examples – volunteered by both mathematicians and math educators – about various mathematical questions. Some examples on the Klein web site so far are:

- How Google works: Markov chains and eigenvalues
- A tale of two triangles: Heron triangles and elliptic curves
- Public key cryptography
- Recurrence and induction
- Map coloring and Gröbner bases
- Matrices and digital images

"This will be a space where teachers and educators and mathematicians do not argue," said Dr. Barton, "– a little playground they all love to be in. It's neutral with respect to school curricula and especially assessment – which is what they argue about most."

Organizers hope for three outputs from the Klein project: a book simultaneously published in several languages, a resource DVD to assist teachers, and a wiki-based web site for contributions. The first Klein Conference was held in Madeira in October 2009 and was followed by half a dozen more at many venues.

Strengthening and extending regional activities

The challenge of a quality mathematics education for all can be met only through collaboration – not just north-south, which is essential, but regionally, to address teaching as it is anchored in contexts and cultures. This will include strengthening existing partnerships and building new ones among foundations, the private sector, academia, government, and like-minded international agencies, such as UNESCO and ICSU. ICMI began to establish such structures decades ago. Beginning with the Inter- American Conferences on Mathematics Education (CIAEM) in Latin America in 1961, many others have been held in Southeast Asia, Anglophone Africa, Francophone Africa, and East Asia.

Reducing the distance between the taught and the new

The mathematical education of teachers is of primary importance, as is the participation of mathematicians in education. For these reasons, the strengthening collaboration between IMU and ICMI is essential. Even as mathematicians become more involved in multidisciplinary projects and other disciplines, there remains the urgent need to maintain the standards of the field itself. In 1990, Jean-Pierre Kahane of France, in his farewell message as president of the ICMI, described his vision of this partnership: "In no other living science is the role of transposition didactique[21] so important at a research level. In no other science, however, is the distance between the taught and the new so large ... In no other science is there such an old tradition of scientists committed to educational questions ... The situation is different now. Mathematics interacts more strongly with other sciences and technologies, mathematicians are looking outside mathematics, many are oriented towards industry, finance, management; the relative importance many place on teaching and on thinking about educational problems is seemingly decreasing. It is time to draw the attention of mathematicians again to educational problems, some of which need to be approached with the extended view of what mathematics is now." One approach to this need is to update curricula in schools to better align with a new world, in which mathematics has progressed into new abstract directions and that, at the same time, uses mathematics in the development of new technologies, industries, and financial tools.

6. The Case for a World Digital Mathematical Library

Like students and scholars in every academic field, those who study mathematics need access to published papers and books, both modern and classic. This need is especially great in the field of mathematics, where current research commonly builds on work published decades or even centuries earlier. For example, one survey found that access to mathematical literature is especially problematic in the developing world, where few institutions can afford expensive professional journals or textbooks; today, mathematics textbooks may retail for more than US $200. Despite well-intentioned efforts to provide books, such as those of UNICEF, outside donors cannot hope to fill the expanding needs of Asia, Africa, and Latin America, where students must rely almost wholly on verbal instruction.

A strategy to improve access to literature

The development of open-source software, e-books, and other Internet innovations, however, suggests a possible solution to this resource crisis: a World Digital Mathematical Library (WDML). Thanks to continued expansion of the Internet to even the smallest countries[22] , such a resource could in theory provide almost every kind of learning resource to students and researchers: classic literature from the nineteenth century and earlier, modern journal articles, books and textbooks, newly posted arXiv preprints, conference proceedings, theses, and more modern forums, such as math blogs, MathOverflow, and social media.

The concept of a WDML is not new; in fact, digital libraries have existed since the early

[21] Didactic transposition refers to the understanding the process of converting factual knowledge into taught knowledge. See, for example, the work of Yves Chevallard. mathematical papers published from 2000 to 2009 cited 665 articles published between 1850 and 1859.

[22] A recent example is the activation of the Eastern Africa Submarine Cable System, which in 2010 brought global service to nine countries, between South Africa and Sudan, that formerly relied on inadequate and expensive satellite connections.

1990s, and most recent mathematical literature is already digitized. But early pioneers of this effort saw how difficult it would be to locate, digitize, link, access, maintain, and finance a worldwide system, and early enthusiasm faded. They realized that a WDML would depend on the leadership of a truly global organization that could act in the common interests of the international mathematical community.

Recently, however, the leaders of the IMU have again recognized that their organization is a logical choice to create a digital library, primarily because of its global reach, wide membership, and tradition of bottom-up, consensual governance. Several recent trends have helped to resurrect the WDML vision, including the growing body of digitized material, the increasing sophistication of online search tools, and the ubiquity of online social media.

The IMU's involvement

In 1998, the IMU Executive Committee created a Committee on Electronic Information and Communication (CEIC) to review the development of electronic information, communication, publication, and archiving so as to keep the EC abreast of current and emerging issues and to advise IMU on all related matters. CEIC was also asked to look into the possibility of a WDML. After several years of study, the CEIC organized a symposium of key leaders from many countries in Washington, DC, in 2012. The Alfred P. Sloan Foundation supported the meeting, and has separately funded a parallel study by the US National Academies on the potential of a WDML. [23]

Attendees described their own current efforts. A summary report by Peter Olver, of the University of Minnesota, reflected the keen interest among mathematicians to raise the project to a high priority. Key leaders from around the world agreed that they saw a window of opportunity, and volunteered their own time and expertise to press ahead expeditiously.

What mathematicians want

Participants at the Washington meeting agreed that a WDML should satisfy several criteria. First, the entire infrastructure should be open and public, including the digital data, the metadata, search algorithms, and all code at the core of the library. It should also be open in its extensibility, thereby inspiring the mathematical community, as well as software engineers and programmers, to develop additional applications and services. There was a strong consensus that the underlying structure be designed as a simple, searchable, and adaptable portal rather than a complex, fixed library: in other words, the WDML should be "future-proof." Curating and maintaining scholarly material should be done locally, by volunteers, rather than in hierarchical fashion. Efficient and low-cost access has the potential to allow talented young mathematicians now languishing in isolated environments to jump-start their careers and bring new contributions to global knowledge.

Barriers and hurdles

The optimism of those attending the CEIC meeting was tempered by their own extensive knowledge of the difficulties that lie ahead. Among these "sticky issues" are the need to define the primary target audience of a WDML and a durable administrative structure that can embrace a diversity of digital communities. A WDML will also face the challenges of coordinating content that is diverse and dispersed; it should develop sophisticated search, interlinking, and referencing capabilities. Useful search techniques will require multilingual and multicultural

[23] The report of this study has since been published and can be obtained from the Arxiv (`arXiv:1404.1905`) or from the US National Academies Press at `http://www.nap.edu/catalog.php?record_id=18619`.

dictionaries and thesauri, including mechanisms for dealing with dynamic and cultural-specific changes in concept, terminology, notation, and rigor. Search algorithms should also be able to determine which parts of a paper are mathematics and which are text or illustration, as well as recognize different layout styles, logical structure, and languages. A particularly thorny issue is that of copyright, especially for current and recent material. Much of the digitized portion of the literature has been formatted by large publishers who need to recoup costs, and by small societies or nonprofits who count on journal income. In addition, some publishers claim copyright on material they digitize even when it is part of the heritage library. Consequently, some participants suggested that a WDML should begin as a modest "World Heritage Digital Mathematics Library" that concentrates at first on freely available literature and serves as a prototype for a broader effort that adds current copyrighted material. One much-discussed idea is that of a "moving wall" for copyrighted material, behind which all mathematical literature becomes freely available as it passes a milestone of five years, or some other agreed upon age. It is important to note that such a moving wall would be of limited benefit to developing countries. The developed countries already have access to the current literature, so the mathematical gap would continue. The IMU advocates more effective mechanisms to provide equitable access to recent publications for developing countries.

A proposed approach

Fortunately, many of these sticky questions have been addressed over a development period of more than a decade by the European DML, or EuDML, which has been cited as a prototype for building a global DML. This encourages many to feel that a WDML is feasible, at least in principle [24] One outline for a WDML, based on the experience of the EuDML, suggests that all mathematical material should be:

- Freely available online, perhaps after an embargo period (the "moving wall")
- Authoritative and enduring
- Augmented continuously with publisher-supplied new content
- Equipped with sophisticated search and interoperability tools
- Developed and organized by a network of institutions

The administrative structure would be a consortium controlled by the international mathematical community, employing a full-time, salaried executive and small staff to oversee the project. An initial step would be to set up a comprehensive registry of all mathematics literature available online. While many tasks can be delegated to volunteers and small stakeholders, the project should be headed by a director whose job depends directly upon its success.

A realistic strategy would be to begin with the classical mathematical corpus which includes mechanisms for scaling up. Decisions about what to include (e.g., what qualifies as "mathematics") can be made by individual content providers, perhaps "softly" regulated by advertising certain best practices (e.g., noting the experience of Math Reviews and Zentralblatt MATH.

A critical component of a WDML will be to provide assistance targeted to the needs of developing countries. For example, few institutions in developing countries have adequate libraries, library infrastructure, or training for librarians. Librarians are low in the academic pecking order, and have little status in negotiating budgets and setting policies. Students have little

[24]See Ulf Rehmann's page on "Retrodigitized Mathematics Journals and Monographs," (http://www.math.uni-bielefeld.de/~rehmann/DML/dml_links.html), which in mid-2012 contained links to 4608 digitized books and 576 digitized journals/seminars.

training in using libraries or searching for online materials.

A few international organizations, such as INASP and HINARI, specialize in providing hands-on coaching for librarians and IT communities in developing countries, and in working with publishers to make scholarly materials available for little or no cost. The IMU can help bolster digital capabilities by partnering with these and other groups, and with publishers wishing to broaden the menu of available mathematical literature. The proposed infrastructure will require start-up funding, along with a modest but indispensable long-term income stream to sustain and continuously upgrade the system. It will need to include librarians expert in the knowledge structure and IT people who maintain the technical structure. There are several logical places to look for upfront funding, and a focused fund-raising effort should begin promptly.

Conclusion

The IMU desires to help make a World Digital Mathematics Library available to the global mathematical community at modest cost. Doing so would help nourish stronger mathematics in developing countries and thereby strengthen the mathematics community as a whole. An equitable WDML would both reflect and extend the IMU's founding credo to "promote international cooperation among mathematicians."

7. A possible roadmap for future action

While the IMU has expanded its activities and goals, it remains a small, volunteer, underfunded organization. In earlier decades, its dues-only structure appeared to be an advantage in some ways, encouraging collaboration and volunteer efforts. But today's global needs – and opportunities – invite a more responsive and dynamic role. Developing countries cry out for assistance as they struggle to build modern mathematics capacity. The "old" IMU could not hope to provide the personal and financial assistance they need to enrich their post-graduation education, pre-service and in-service training for teachers, and the broader educational base on which to build a mathematically literate work force and society. The IMU may choose to develop a strategy designed to attempt this important and challenging new role.

In most developing countries, mathematical development is limited by a small population of qualified educators. More participants are needed at virtually every level: primary and secondary school teachers; faculty at the undergraduate, masters and especially the PhD levels; research mathematicians, who will contribute to a thriving mathematical environment that will keep mathematical talent in the country, counteracting the brain drain, and those who can foster expertise in modeling, interdisciplinary topics, and industrial mathematics. With few professors in place to train the next generation of teachers, researchers, and faculty, developing countries cannot harness the power of mathematics to improve food security, combat diseases, use information tools, and power the economy.

The IMU and its more than 70 member countries are committed to building a global mathematical capacity that is based on three platforms: quality mathematics education for all, a new emphasis on mathematics in developing countries, and open access to current and archival mathematics literature.

A new strategy might encompass the IMU's traditional activities, including the International Congress of Mathematicians and the awarding of the Fields and other medals, along with new activities targeted at the needs of developing as well as developed societies. A central element of this vision, though built on past custom, is the tradition of volunteerism, on which many of the programs listed above depend. These include the CANP, the Klein Project, and the "Experiencing Mathematics" exhibition, all of which require the leadership of both the mathematics and mathematics educator communities throughout the world.

This strategy also depends on continuing shared efforts between the IMU and ICMI, which have grown closer during recent years. The two organizations, by attending one another's congresses, jointly designing new programs, and expanding their efforts to raise funds for desirable programs, have the potential to achieve even greater results than have been possible in the past.

As global organizations, they can also call on others for help: governments, funding agencies, regional organizations, professional societies. Using their own outreach projects as models, they can translate what they have learned to regional, national, and global extensions of these models that are designed and managed by mathematicians and math educators who are experts on the culture and particular needs of their environment.

Among the many objectives of this partnership, the following should command a high priority. These objectives are cast as suggestions for those of the IMU and ICMI constituencies which are best positioned to carry them out.

• Strengthen education at every level

IMU members generally agree that educational improvement is an overwhelming need facing the mathematics enterprise of developing and developed countries alike. In most developing countries, mathematical capacity is limited by low numbers of well-qualified educators. More participants are needed at virtually every level: qualified primary and secondary school teachers; faculty at the undergraduate, masters and especially the PhD levels; and research mathematicians, including those with expertise in modeling, interdisciplinary topics, and industrial mathematics. Stronger education is also an urgent need for most developed countries if they are to meet the ever-expanding demands of mathematics-based sciences and technologies.

• Incorporate fund-raising into the IMU mission

The 2006 IMU General Assembly proposed that the incoming Executive Committee identify a site for a stable office and an expanded menu of IMU activities, including a fund-raising expertise commensurate with the desire of the Union to meet the new requests and demands coming from around the world. A permanent office location has now been established in Berlin, Germany, along with the IMU archives. From this base, IMU can aspire to the kind of resource building that can support not only existing outreach programs, which are modest in scope, but the innovative and ground-up approaches required to make the Union's activities even more effective on a larger scale.

• Support regional networks

In addition to revitalizing individual universities and other institutions, our advisers favor the use and improvement of networks of institutions. Networks offer many advantages, such as
(1) creating the critical masses of students and researchers that reduce professional isolation;
(2) increasing program flexibility by making more diverse skills available to participants, and
(3) allowing students and organizations to share infrastructure, mentors, and career skills.

Regional networks bring several advantages for mathematicians. They are relatively inexpensive, since many of their activities are generated by people and institutions already in place. They rely on local people, an essential ingredient for success. They increase the magnitude and power of projects by adding the expertise of multiple nodes. They allow students access to more role models, partners, and career possibilities than are visible from a single location. And they add the authority of increased mass to each of the member nodes, allowing mathematicians to speak with a more powerful and unified voice.

• Encourage capacity building in situ

The intellectual capacity of many developing countries has been weakened in recent decades by brain drain to the scientifically more advanced countries. When the number of trained mathematicians is small, as it is in so many developing countries, the negative effect of losing even a single brilliant mathematician can be profound when one considers the loss of teaching, mentoring, and partnering activities as well as research. By encouraging and providing incentives for bright young mathematicians to study at home or at least return there after intensive "sandwich" programs abroad, the IMU can help build enduring strength in the home country. A population of locally employed mathematicians is needed to prepare young scholars not only for academia but also for technical jobs in government and the private sector.

• Develop a Worldwide Digital Mathematics Library

As discussed in Section 6, open access to mathematics literature can be an enormous step forward for developing countries, which seldom have the journals or textbooks they need. Mathematics students and faculty alike need to be connected to the literature of their field. The IMU has recently begun a strong effort to develop a Worldwide Digital Mathematics Library, which has the potential to break through to otherwise isolated countries and allow them to leap-frog ahead by gaining new knowledge and an appreciation for world-wide events in their fields.

• Work toward a larger role for industry

Most mathematicians agree that all mathematical topics share the same continuum, while some of them are described in terms of their "pure" or "applied" nature as a matter of convenience. Many activities bridge such differences, as fundamental work gives rise to new applications, and applications commonly inspire new fundamental work. While industry well appreciates the need for applied mathematical research, and supports its own applied programs, stronger collaboration between fundamental mathematicians in academia and those in industry is a desirable goal for both sides. With the rapid growth of information technology, modeling, data handing, bioinformatics, and other math-based fields, the extension of mathematics into interdisciplinary efforts will increasingly drive social and economic progress in the future. The IMU, spanning both fundamental and applied activities, can do more to help bridge the communication gap and invite more participation by industry in the activities of the Union.

• Improve the public image of mathematics

A serious impediment to stronger mathematical capacity is its poor public image. Young people get little support or encouragement to enter the field, often lacking inspiring teachers and learning little about the exciting math-based careers that will require more talented entrants in the years ahead. The IMU and ICMI both have the opportunity to do far more in communicating the value of mathematics to the general public and demonstrating its importance to science, culture, and economic development. Teachers and researchers alike can do far more via public talks, writings, and TV presentations to explain the needs for their subject in modern life. They can also reach out to the private sector to demonstrate the value of scientific thinking for firms small and large

• Make mathematics teaching more relevant and more exciting

The IMU and ICMI can do much more to encourage better teaching. Beginning in primary school, teaching should move beyond traditional emphases on memorization and computation to include practical exercises and a vision of the broader aspects of mathematics. To achieve this requires teachers and professors who have been exposed to the exciting problem-solving potentials of mathematics. Even in the less-developed countries, teachers who have access to basic tools and information sources, including functional computers, software, and online journals, can absorb some of this excitement and impart the flavor of modern mathematics to their students[25] . Faculty motivation should include promotion on the basis of merit rather than the common criterion of seniority. In addition, modern teaching of applied and industrial mathematics should accompany the teaching of pure mathematics as a focus of every institution.

- **Promote a more active role for governments**

The IMU has deliberately assumed a non-political role in its activities, taking care not interfere in government policies. Today, however, the Union has the opportunity to do more in advising governments on the need for support of mathematical research and education, and on best practices appropriate to their own educational levels. Because salaries, scholarships, and much capital spending in developing countries are determined by national governments, political leaders are well positioned to accelerate mathematical development. They also have the responsibility to do so in an era when every country can benefit from increased mathematical and technical skills among its people. The IMU can now take advantage of its carefully developed reputation for neutrality by advising governments on the enormous returns to be gained by investing in the mathematics education of their young people.

All these objectives would have been difficult, if not impossible, for the "old" IMU to contemplate, with its minimal resources and limited reach. They will still be difficult, just as mathematics and math education will always be challenging endeavors. But there are reasons to view the moment as propitious for an expanded agenda. The Internet has the potential to remove the problem of access to information and literature, bringing instead the news of great opportunities for those with proficiency in the mathematics-based skills of science, engineering, and technology. Governments in the developing world are more aware of the need to build their capacity in STEM-based teaching and learning, even while not enough of them have the resources to move rapidly. And mathematics itself is proving not only to be central to the fast-growing interdisciplinary fields of science and engineering, including the biosciences and materials research, but also undergoing exciting growth in many of its sub-disciplines, pure as well as applied.

Just as the world is beginning to appreciate the pervasive role of mathematics, the IMU has begun to articulate its own evolving role – one that is truly world-wide. Just as the world already benefits from the capabilities of mathematicians in every region, the members of the IMU now see the opportunity to build on this reality, communicate it to policy makers and the general public, and further encourage the development of mathematical talent everywhere. Only when this talent is allowed to grow and flourish across the globe will its inhabitants have the full range of abilities to address the worldwide challenges that lie ahead.